

# 超新星残骸の衝撃波と粒子加速の理論的進展 ( Theoretical progress of SNR shocks and particle accelerations )

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# Nonthermal Universe

Earth's bow shock

Solar flare

Interplanetary shock

Colliding wind binary

Nova

Supernova remnant

Micro quasar

Pulsar and Pulsar wind nebula

Magnetar

Gamma ray binary

Superbubble

Cluster of galaxies

Starburst galaxies

Active galactic nuclei

Gamma ray burst

And so on.

**These objects have nonthermal particles**

**What is the acceleration mechanism?**

**What is the origin of CRs ?**

# Galactic cosmic ray

It is believed that the source of the Galactic CR is SNR.

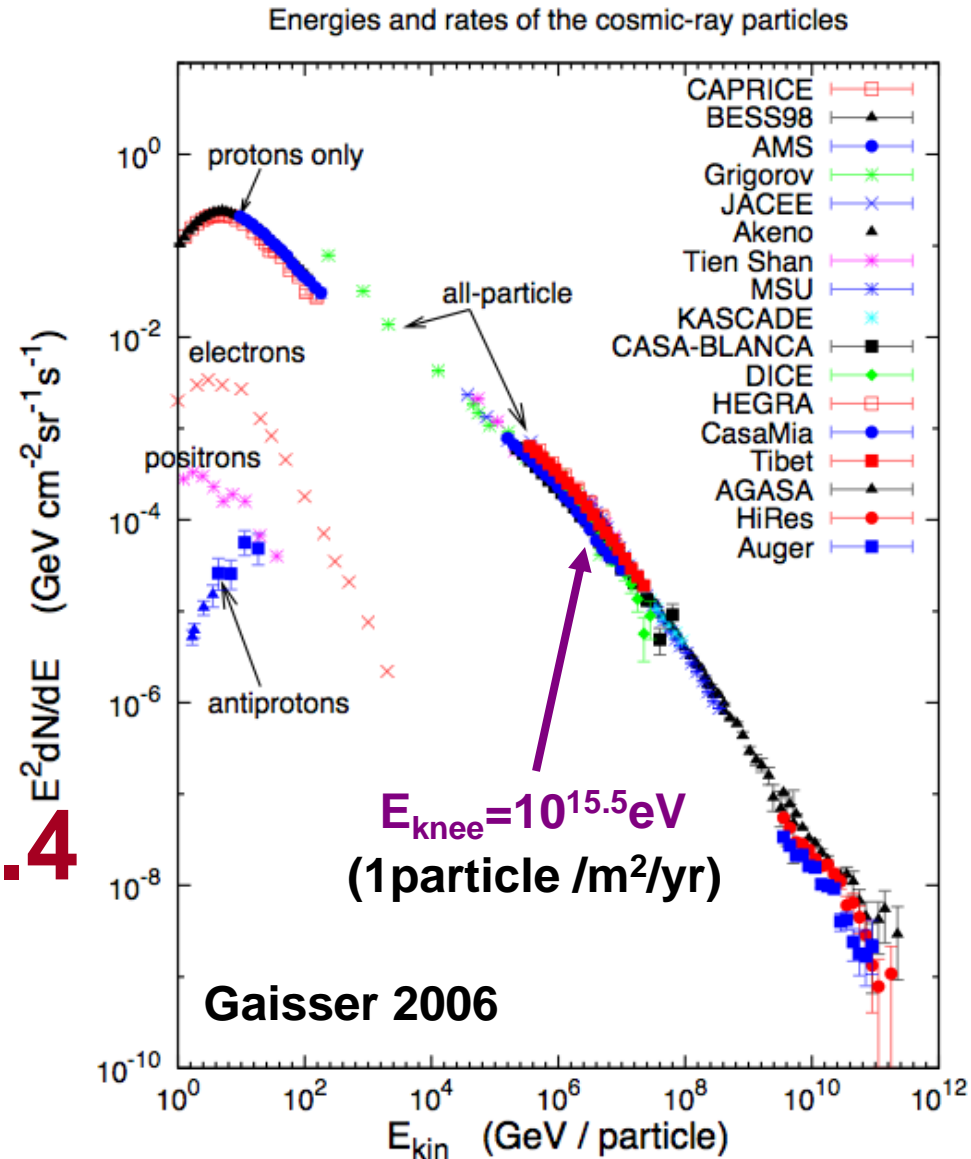
Observed spectrum

$$N(E) \propto E^{-2.7} \quad (E < 10^{15.5} \text{eV})$$

Taking into account the propagation effect

$$Q_{\text{sour}} \propto E^{-2.1-2.4}$$

The source of UHECRs has not been understood.



# Galactic diffusion of cosmic ray

## Leaky box model

Steady state

$$\frac{dN_{\text{CR}}(E)}{dt} = -\frac{N_{\text{CR}}(E)}{t_{\text{esc}}(E)} + Q_{\text{sour}}(E) \longrightarrow N_{\text{CR}} = t_{\text{esc}}(E) Q_{\text{sour}}(E)$$

$$t_{\text{esc}}(E) = L_{\text{size}}^2 / D_{\text{diff}}(E) , \quad D_{\text{diff}}(E) \propto E^{\delta} , \quad Q_{\text{sour}}(E) \propto E^{-s}$$

$$N_{\text{CR}}(E) \propto E^{-(s+\delta)}$$

$$\text{B/C obs. : } \delta = 0.3 - 0.6$$

$$\text{CR obs. : } s + \delta = 2.7$$

$$\Rightarrow s = 2.1 - 2.4$$

# Particle Accelerations

**Diffusive shock acceleration (1st order Fermi)**

**Turbulent acceleration (2nd order Fermi)**

**Shear acceleration**

**Surfing acceleration**

**Shock drift acceleration**

**Acceleration in the magnetic reconnection**

**Kinetic Alfvén acceleration**

**Direct acceleration by the electric field**

**And so on.**

# **Diffusive shock acceleration(DSA)**

**Due to the simple theory,**

**the DSA has been most studied in detail.**

**However,**

**Non-relativistic shock or Relativistic shock**

**e-/e+ plasma or e-/ion plasma**

**or e-/e+/ion plasma**

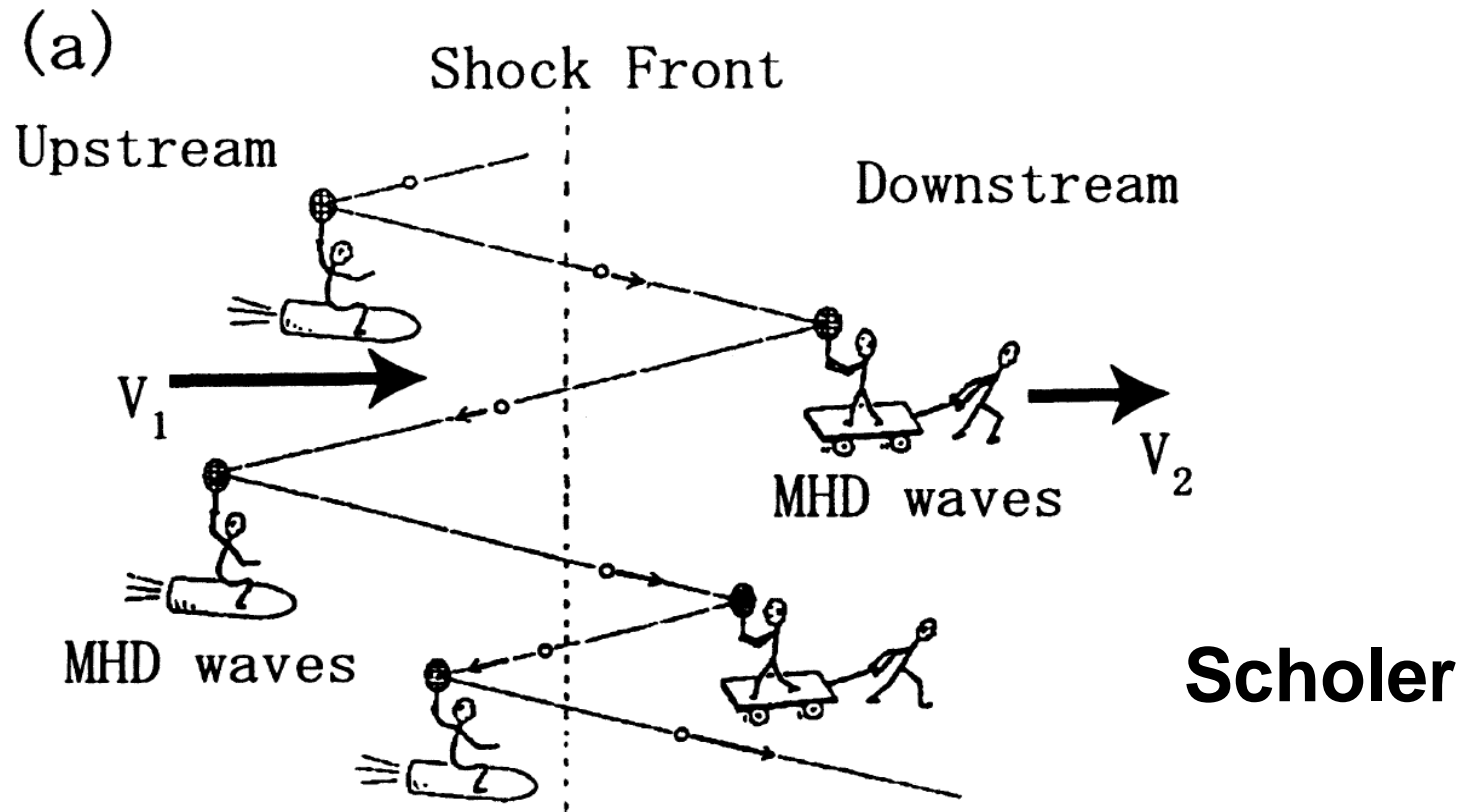
**Strongly magnetized plasma or Weakly magnetized plasma**

**Test particle DSA or Nonlinear DSA**

**Fully ionized plasma or Partially ionized plasma**

**There are many types of shocks.**

# Diffusive Shock Acceleration(DSA)

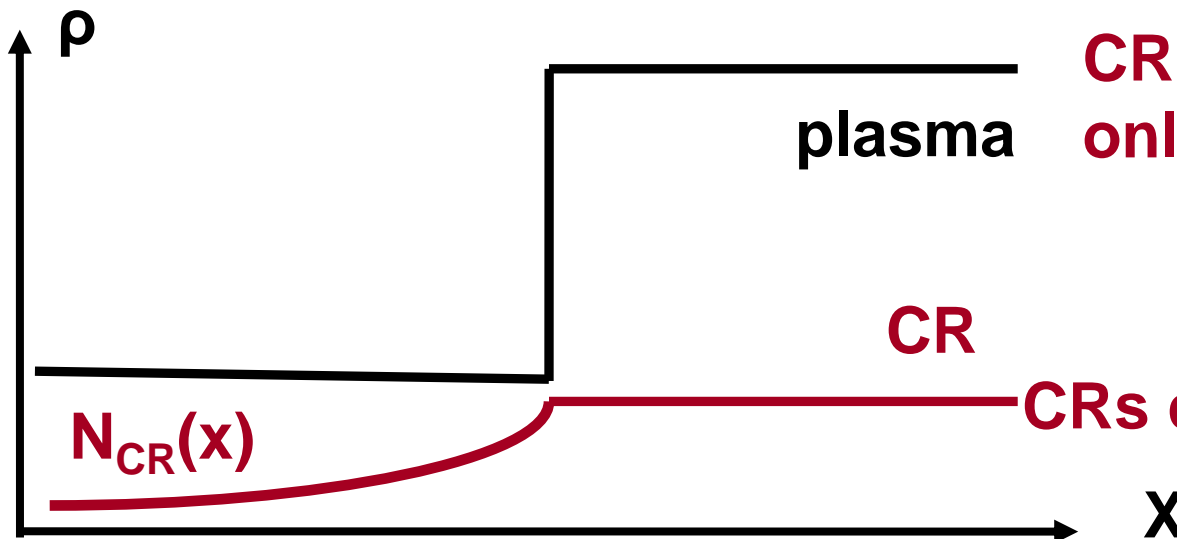
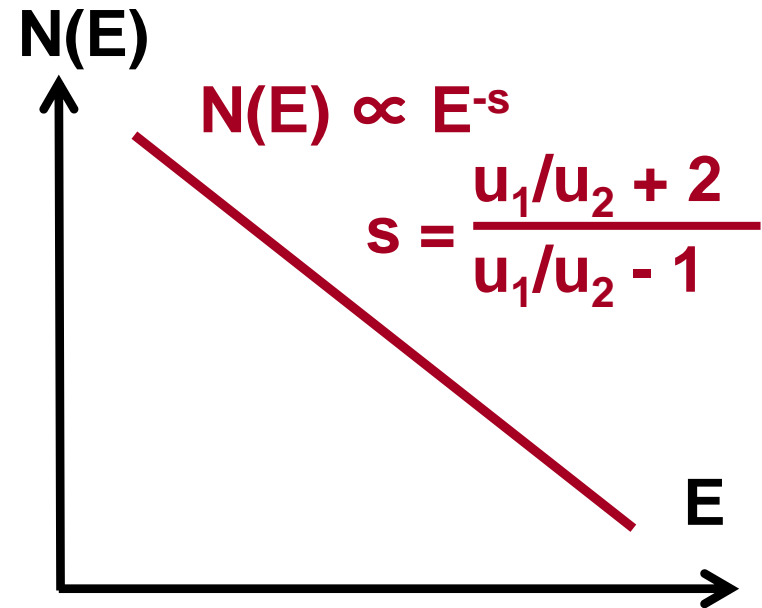
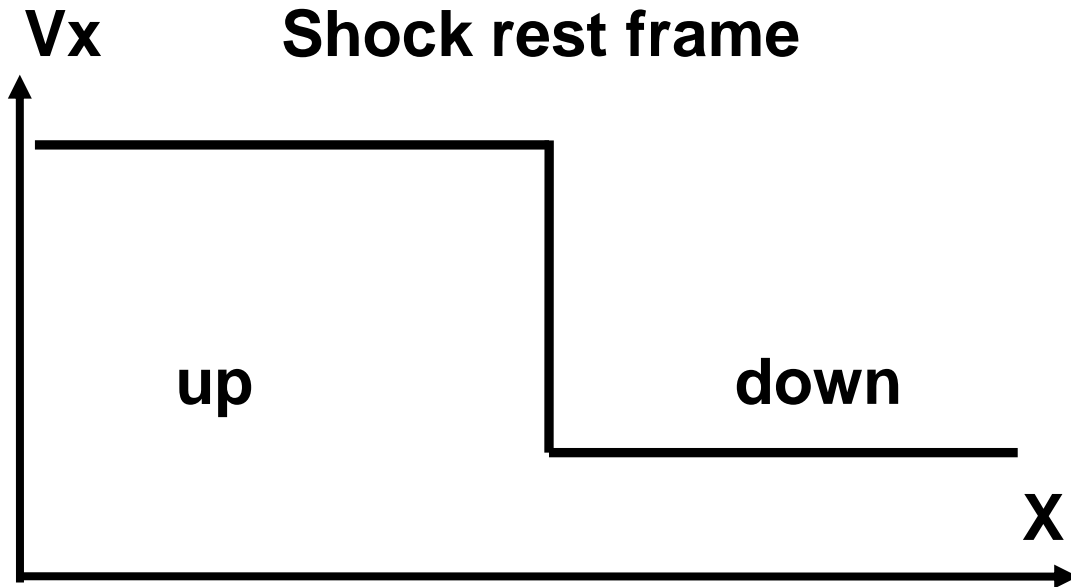


CRs are scattered by MHD waves.

CRs excite the MHD waves.

Axford 1977, Krymsky 1977, Blandford&Ostriker 1978, Bell 1978

# Non-relativistic shock acceleration



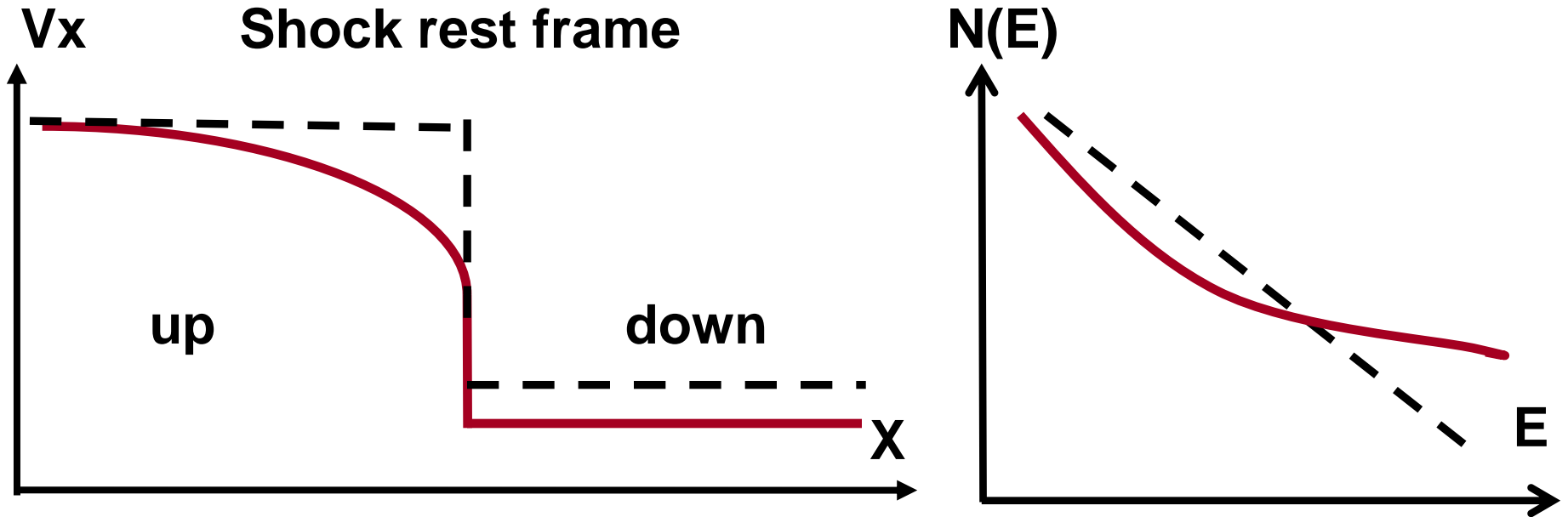
CR spectrum depends on only the velocity jump.

$$M \gg 1, N(E) \propto E^{-2}$$

CRs diffuse to the upstream.



# Nonlinear DSA Model



The upstream plasma is pushed by the CR pressure.

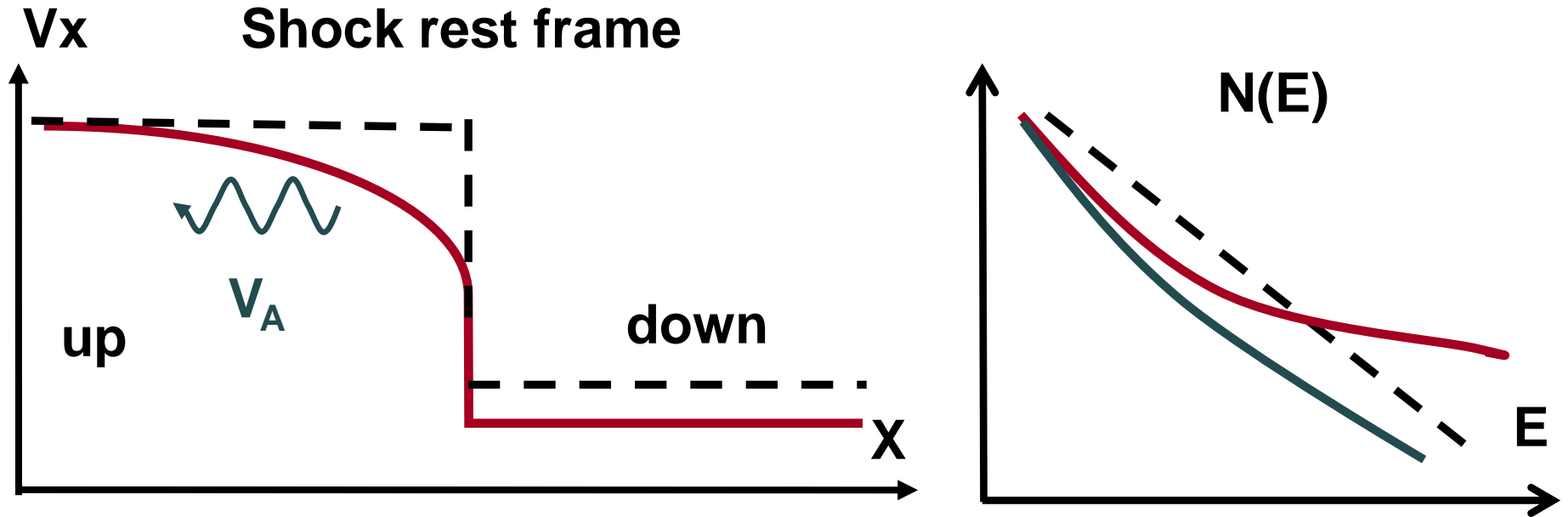
The total compression ratio becomes large.

The CR spectrum becomes harder than  $E^{-2}$ .

This is inconsistent with Galactic CR observations.

e.g., Drury & Volk (1981), Malkov & Drury (2001)

# Recent Nonlinear DSA Model



Waves excited by CRs go to the upstream.

The velocity of scattering centers is  $V_1 - V_A$ .

$V_A$  is comparable to  $V_1$  because of the strong magnetic field

The jump of the scattering velocity becomes small.

As the result, the CR spectrum becomes steep.

e. g., Ptuskin & Zirakashvili (2008)

# Maximum Energy of Accelerated Particles

$$t_{\text{acc}} \sim \frac{3D_{\text{Bohm}}}{u_1^2}$$

Drury (1983)

To accelerate to the high energy,  
we need a finite time.

$$D_{\text{Bohm}} = \frac{cE}{3eB}$$

$$t_{\text{acc}} = t_{\text{age}} \rightarrow \text{Age limited } E_{\text{max}}$$

CRs should be confined in the acceleration region.

$$L_{\text{diff}} = D/u_{\text{sh}} = L_{\text{size}} \rightarrow \text{Escape (Size) limited } E_{\text{max}}$$

energy losses (cooling) should be unimportant.

$$t_{\text{acc}} = t_{\text{cool}} \rightarrow \text{cooling limited } E_{\text{max}}$$

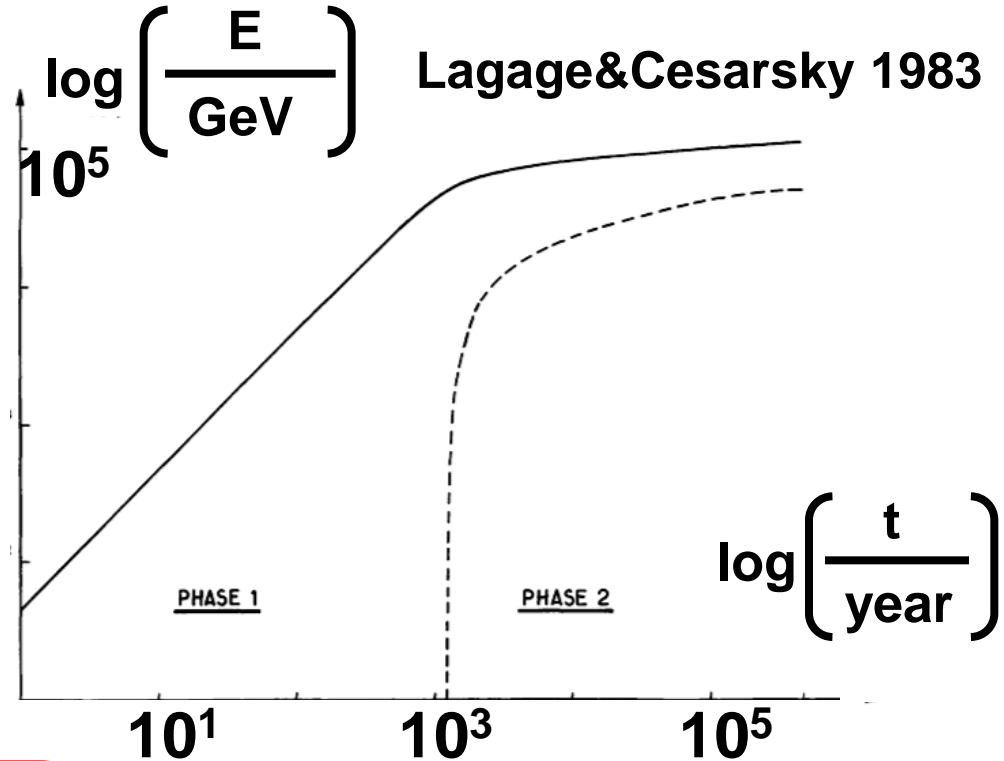
# Age-limited maximum energy in SNRs

## Age-limited model

$$\frac{dE}{dt} = \frac{E}{t_{\text{acc}}} \propto u_1^2 \propto \begin{cases} t^0 \\ t^{-6/5} \end{cases}$$

$$t_{\text{acc}} = \frac{3D_{\text{Bohm}}}{u_1 - u_2} \left( \frac{1}{u_1} + \frac{1}{u_2} \right)$$

$$D_{\text{Bohm}} = \frac{c^2 E}{3eB}$$



**$E \sim E_{\text{max}}$  at  $t_{\text{Sedov}}$**

$$E_{\text{max}} \sim 10^{14} \text{ eV} < 10^{15.5} \text{ eV}$$

If  $B$  is amplified,  $E_{\text{max}} \sim E_{\text{knee}}$

$$E_{\text{SN}} = 5 \times 10^{51} \text{ erg}$$

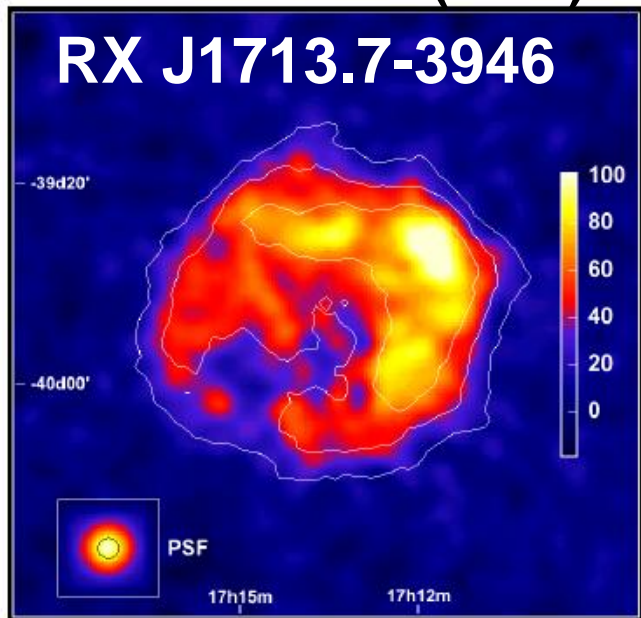
$$M_{\text{ej}} = 0.5 M_{\text{solar}}$$

$$B = 1 \mu\text{G}$$

$$n_{\text{ISM}} = 3 \times 10^{-3} \text{ cm}^{-3}$$

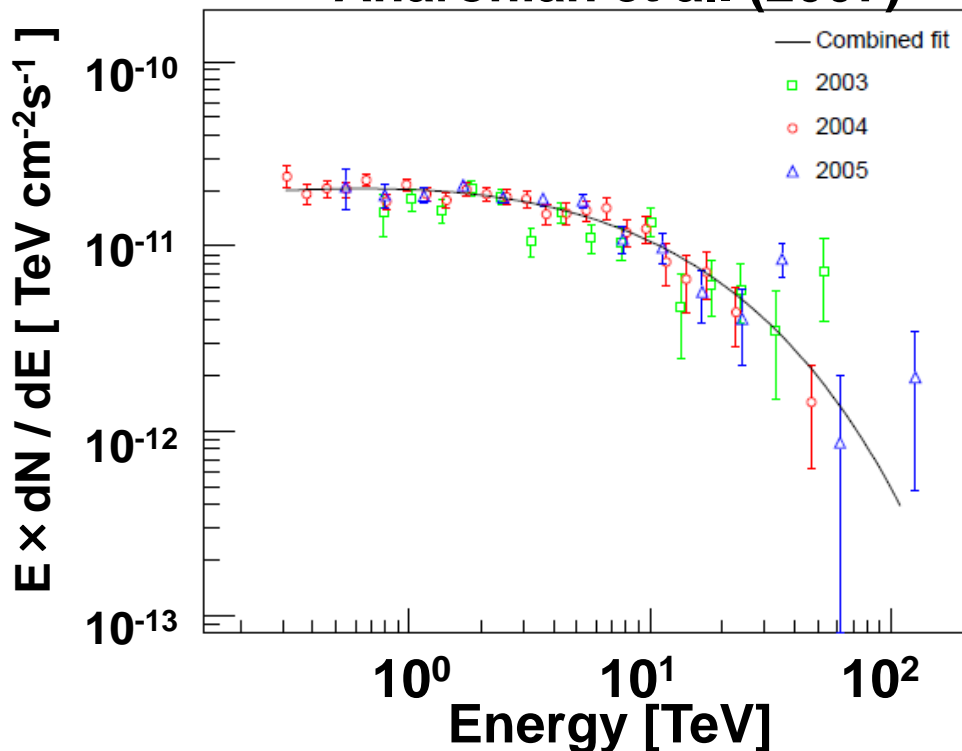
# $\gamma$ -ray spectrum of young SNRs

Aharonian et al. (2006)



TeV  $\gamma$ -ray image

Aharonian et al. (2007)



$\gamma$ -ray spectrum starts to decrease at 10TeV.

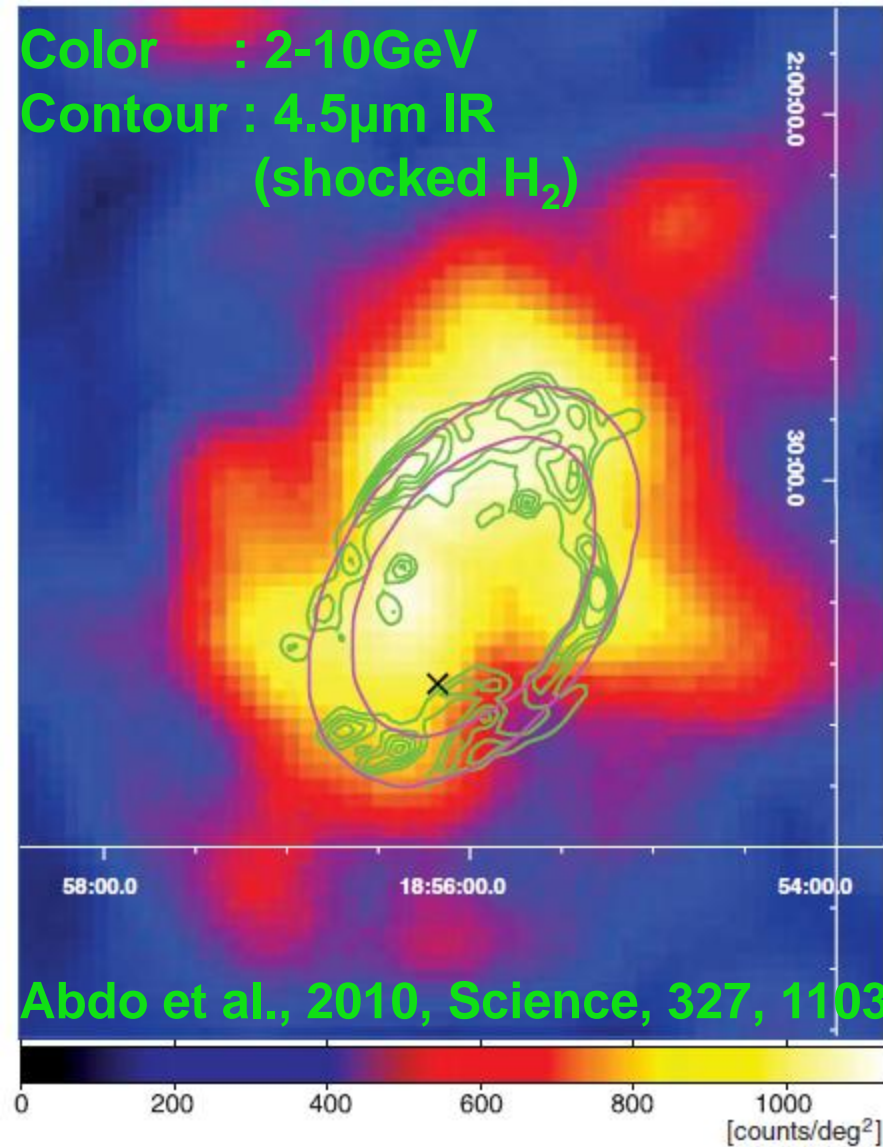
If the  $\gamma$  ray is due to the pion decay,  $E_{\text{max}} \sim 10^{14} \text{eV} < E_{\text{knee}} = 10^{15.5} \text{eV}$

Even though  $t_{\text{age}} \sim 1600 \text{yr} \sim t_{\text{Sedov}}$ ,  $E_{\text{max}} < E_{\text{knee}}$  ?

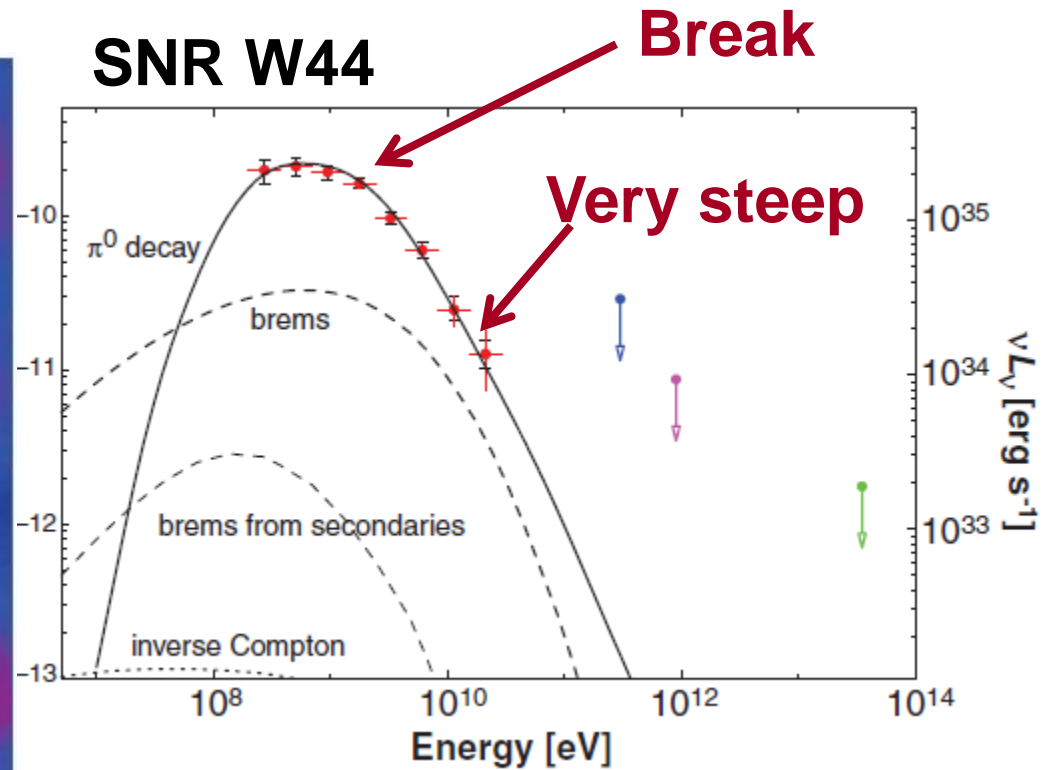
Radio and X-ray observations show that

high energy  $e^-$  are also accelerated.

# $\gamma$ -ray spectra of Middle-aged SNRs



SNR W44



$t_{\text{age}} \sim 2 \times 10^4 \text{ yr}$

$E_{\text{br}} \sim 9 \text{ GeV}$

$d \sim 3 \text{ kpc}$

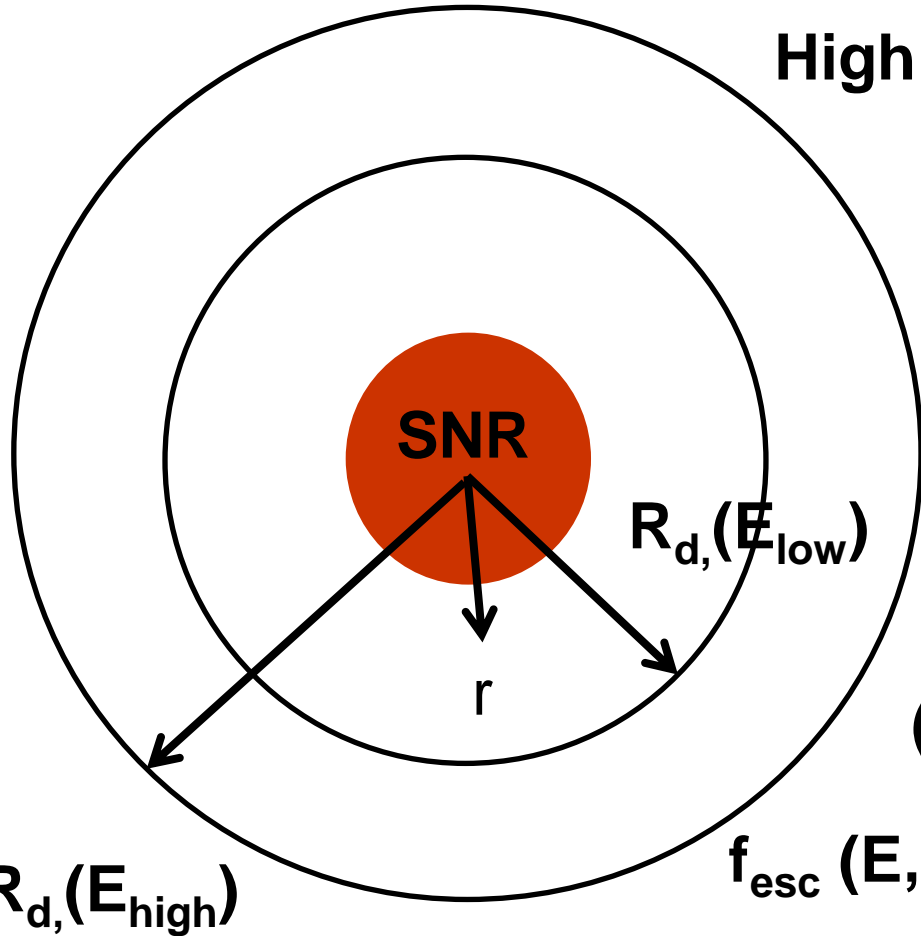
$s_1 \sim 2.0$

$s_2 \sim 3.3-3.7$

Break , very steep spectrum

# Escape of Cosmic Rays

High energy CRs diffuse faster than low energy CRs



$$R_d(E) = [4D(E)t]^{1/2} \propto E^{0.5\delta}$$

$$D(E) = \chi \left[ \frac{E}{10\text{GeV}} \right]^\delta \text{cm}^2 \text{s}^{-1}$$

$$\delta > 0$$

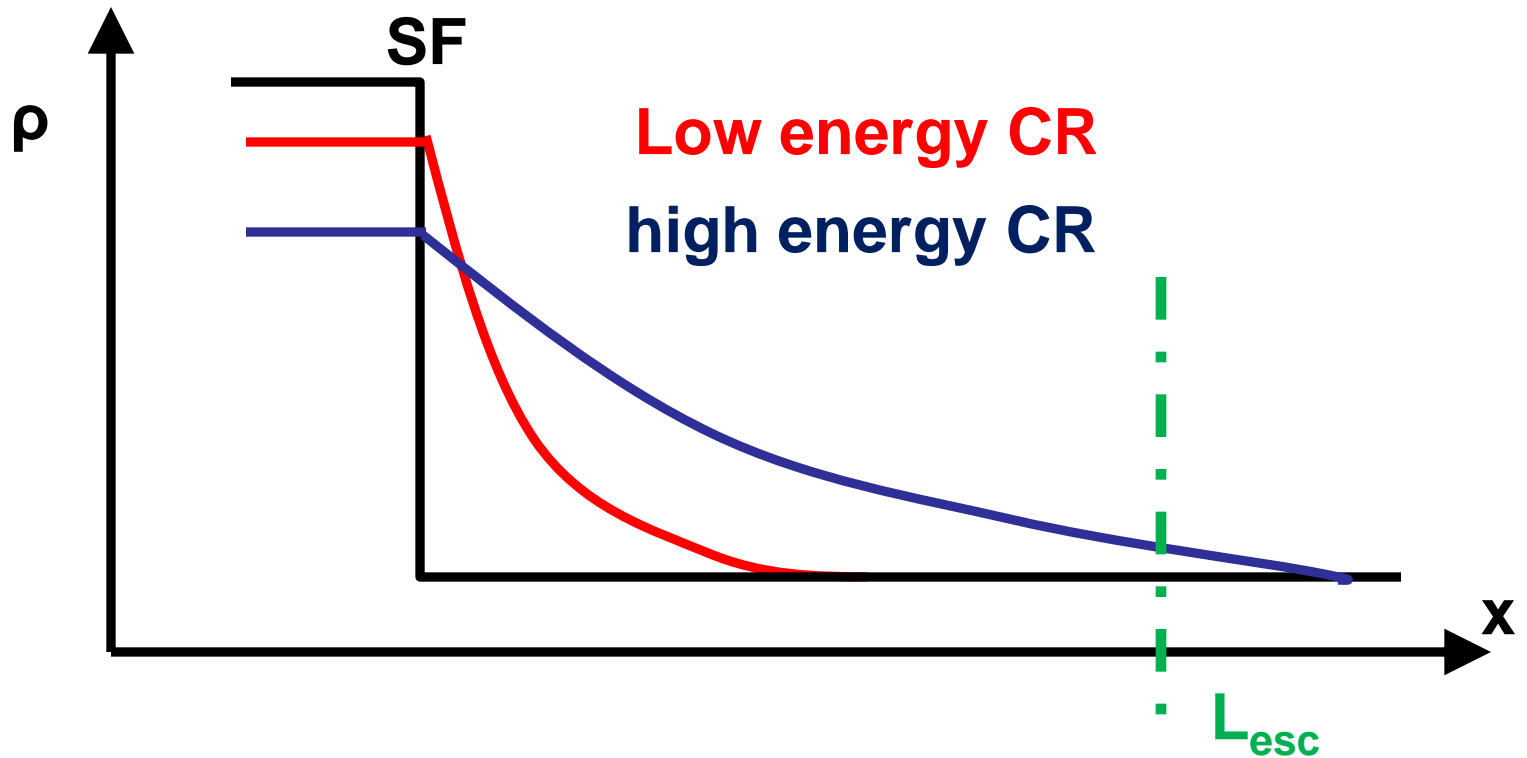
(Galactic mean value  $\delta=0.3-0.6$ )

$$f_{esc}(E,r) \propto Q_s(E) / R_d(E)^3 \times \exp[-(r/R_d)^2]$$

$$\propto Q_s(E) E^{-1.5\delta}$$

$f_{esc}(E)$  is softer than  $Q_s(E)$ . Aharonian & A toyan(1996), Gabici et al (2009)

# Escape-limited model



$$L_{diff} = D(p) / u_{sh}$$

Escape boundary

$$\text{Escape condition : } L_{diff} = L_{esc}$$

Higher energy CR escapes at earlier epoch.



# Escape-limited maximum energy

Free expansion phase (  $t < 200\text{ yr}$  ): age limited

$$E_{\text{max}} \propto t \quad , \quad E_{\text{max}} ( t_{\text{Sedov}} ) = E_{\text{knee}} \quad (\text{B should be amplified})$$

Sedov phase (  $t < 10^5\text{ yr}$  ) : escape limited

$E_{\text{max}}$  is decided by  $L_{\text{diff}} = L_{\text{esc}}$

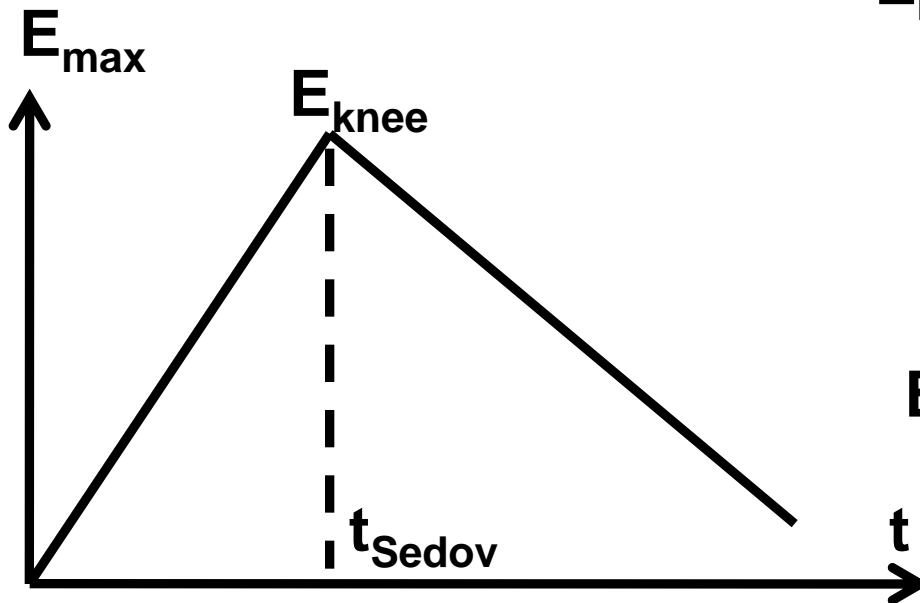
$$L_{\text{diff}} = D / u_{\text{sh}}$$

$$L_{\text{esc}} = \kappa R_{\text{sh}}$$

$$E_{\text{max}} \propto \kappa B R_{\text{sh}} u_{\text{sh}} \propto \kappa B(t) t^{-1/5}$$

(  $R_{\text{sh}} \propto t^{2/5}$  )

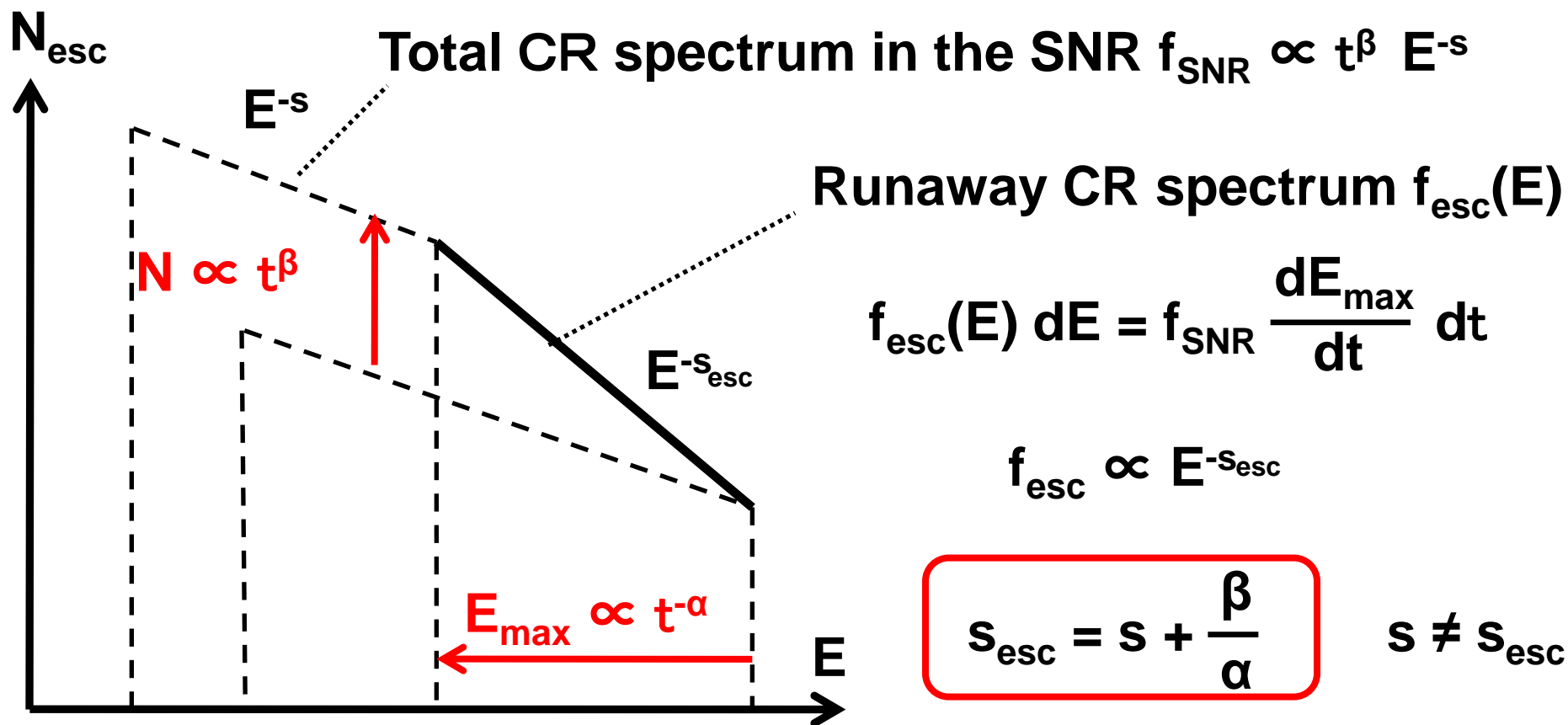
$E_{\text{max}}$  decreases with time



# Spectrum of runaway CR

CR number  $N(E=mc^2) \propto t^\beta$  ,  $\beta \geq 0$

Maximum momentum  $E_{\max} \propto t^{-\alpha}$  ,  $\alpha > 0$



# Observed spectrum and spectrum at shocks

Leaky box model

Steady state

$$\frac{dN_{\text{CR}}(E)}{dt} = -\frac{N_{\text{CR}}(E)}{t_{\text{esc}}(E)} + Q_{\text{sour}}(E) \longrightarrow N_{\text{CR}} = t_{\text{esc}}(E) Q_{\text{sour}}(E)$$

$$t_{\text{esc}}(E) = L_{\text{size}}^2 / D_{\text{diff}}(E), \quad D_{\text{diff}}(E) \propto E^{\delta}, \quad \text{ ~~} Q_{\text{sour}}(E) \propto E^{-s}~~$$

$$Q_{\text{sour}}(E) \propto E^{-s_{\text{esc}}}$$

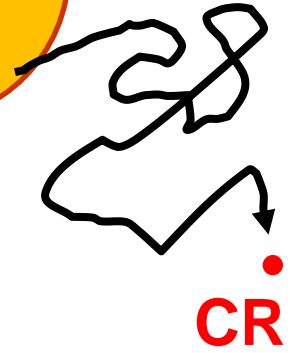
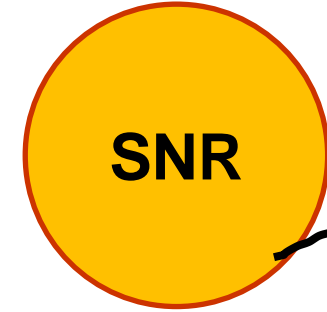
$$N_{\text{CR}}(E) \propto E^{-(s + \frac{\beta}{\alpha} + \delta)}$$

$$\text{CR obs. : } s + \frac{\beta}{\alpha} + \delta = 2.7$$

$$\delta = 0.3 - 0.6$$

# Runaway CR distribution around an SNR

$$\frac{\partial}{\partial t} f_{\text{CR}}(t, r, E) = D(E) \Delta f_{\text{CR}}(t, r, E) + Q_{\text{sour}}(t, r, E)$$



CRs escape from SNR surface  $R_{\text{esc}}(E)$

$$Q_{\text{sour}}(t, r, E) = \frac{N_{\text{esc}}(E)}{4\pi r^2} \delta(r - R_{\text{esc}}(E)) \delta(t - t_{\text{esc}}(E))$$

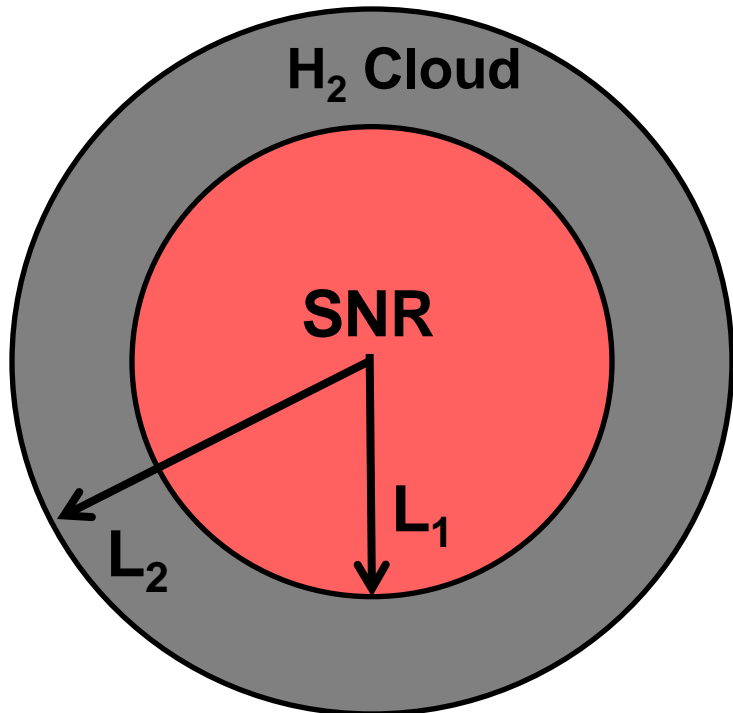
The solution to the diffusion equation is

$$f_{\text{CR}}(t, r, E) = \int d^3r' f_{\text{point}}(t, |\vec{r} - \vec{r}'|, E) \frac{\delta(|\vec{r}'| - R_{\text{esc}}(E))}{4\pi |\vec{r}'|^2}$$

$$= \frac{\exp\left\{-\left(\frac{r - R_{\text{esc}}}{R_d}\right)^2\right\} - \exp\left\{-\left(\frac{r + R_{\text{esc}}}{R_d}\right)^2\right\}}{4\pi^{3/2} R_d R_{\text{esc}}} N_{\text{esc}}(E)$$

# CR spectrum $F(E)$ in $L_1 < r < L_2$

$$\begin{aligned}
 F(E) &= \int_{L_1}^{L_2} f_{\text{ext}}(t_{\text{age}}, r, p) 4\pi r^2 dr \\
 &= \frac{N_{\text{esc}}(p)}{2} \left\{ \frac{R_d}{\sqrt{\pi} R_{\text{esc}}} \left( e^{-\left(\frac{L_1 - R_{\text{esc}}}{R_d}\right)^2} - e^{-\left(\frac{L_2 - R_{\text{esc}}}{R_d}\right)^2} - e^{-\left(\frac{L_1 + R_{\text{esc}}}{R_d}\right)^2} + e^{-\left(\frac{L_2 + R_{\text{esc}}}{R_d}\right)^2} \right) \right. \\
 &\quad \left. + \text{erf}\left(\frac{L_2 - R_{\text{esc}}}{R_d}\right) - \text{erf}\left(\frac{L_1 - R_{\text{esc}}}{R_d}\right) + \text{erf}\left(\frac{L_2 + R_{\text{esc}}}{R_d}\right) - \text{erf}\left(\frac{L_1 + R_{\text{esc}}}{R_d}\right) \right\}
 \end{aligned}$$



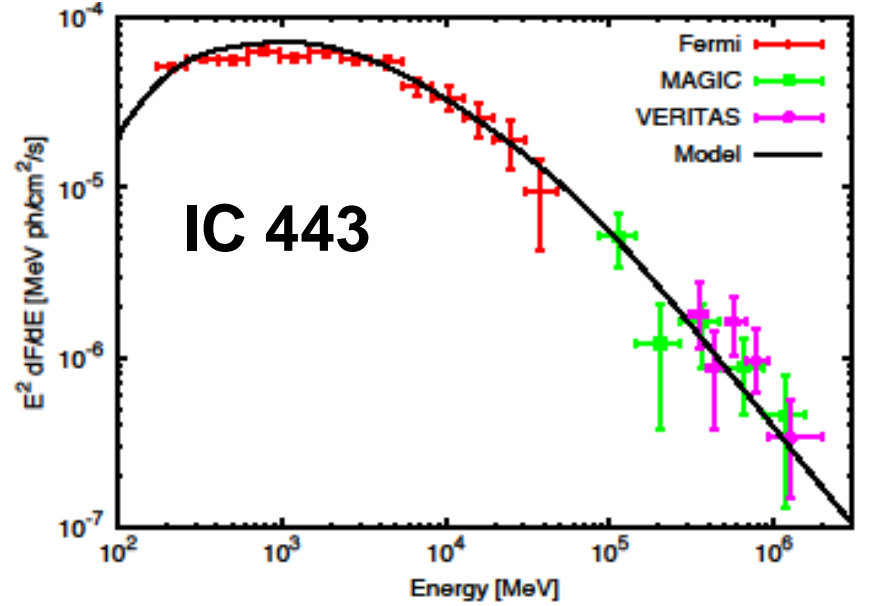
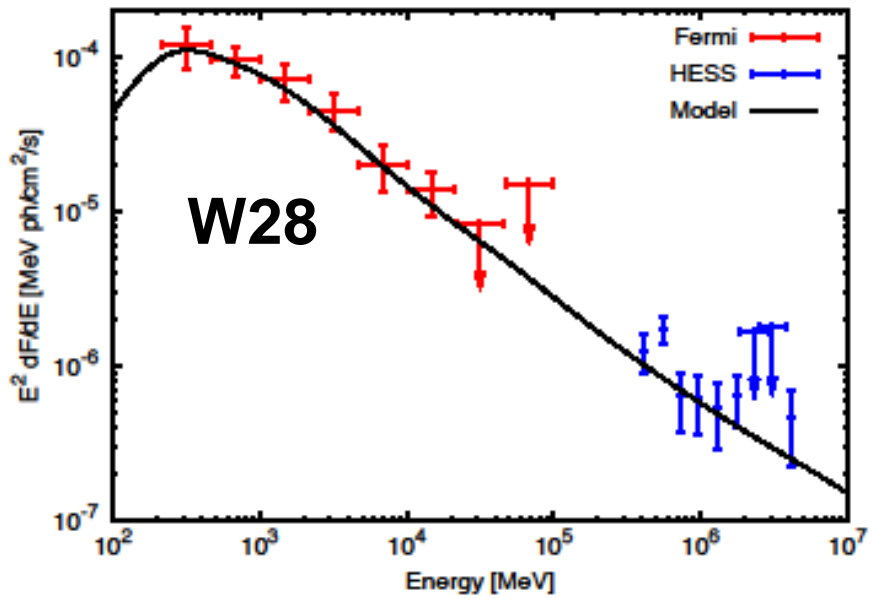
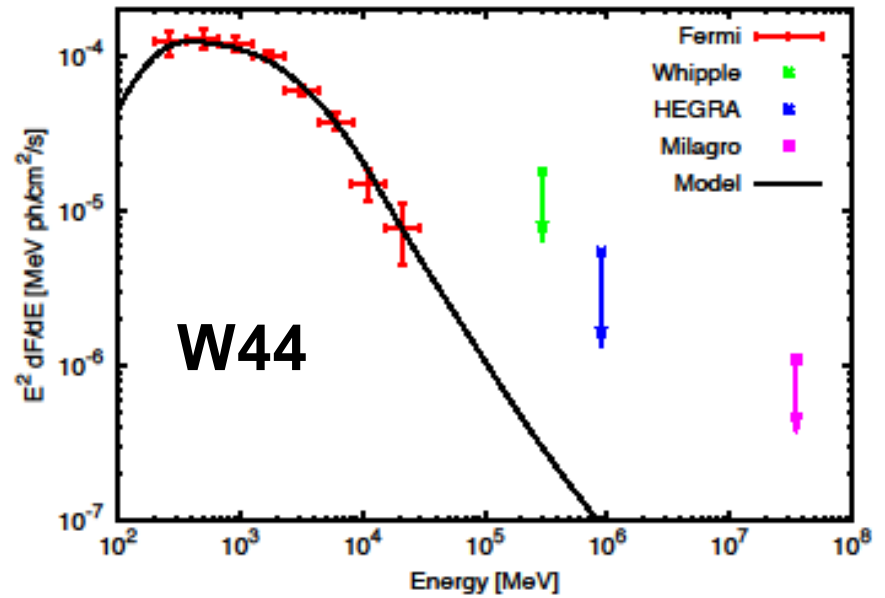
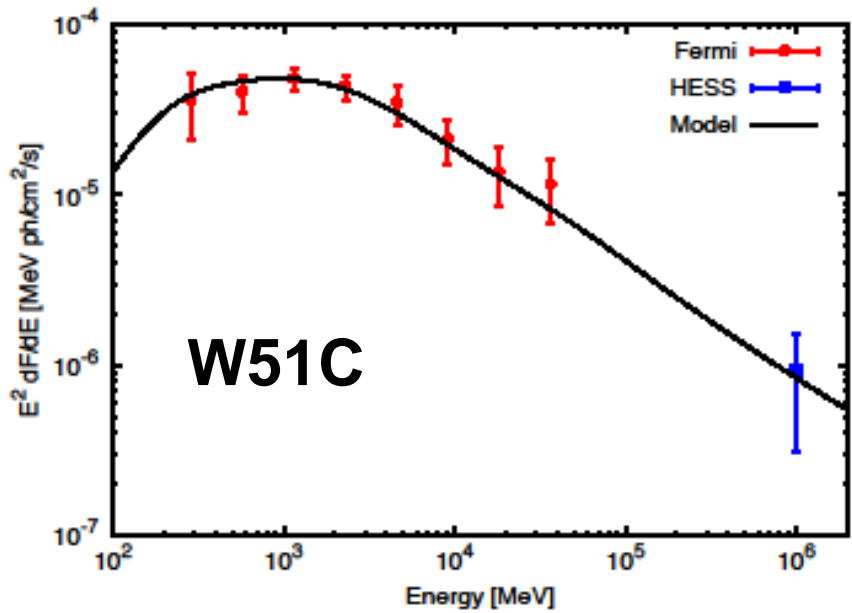
$$\text{erf}(x) = \frac{2}{\pi^{1/2}} \int e^{-y^2} dy$$

$$R_d(E) = [4D(E)(t - t_{\text{esc}}(E))]^{1/2}$$

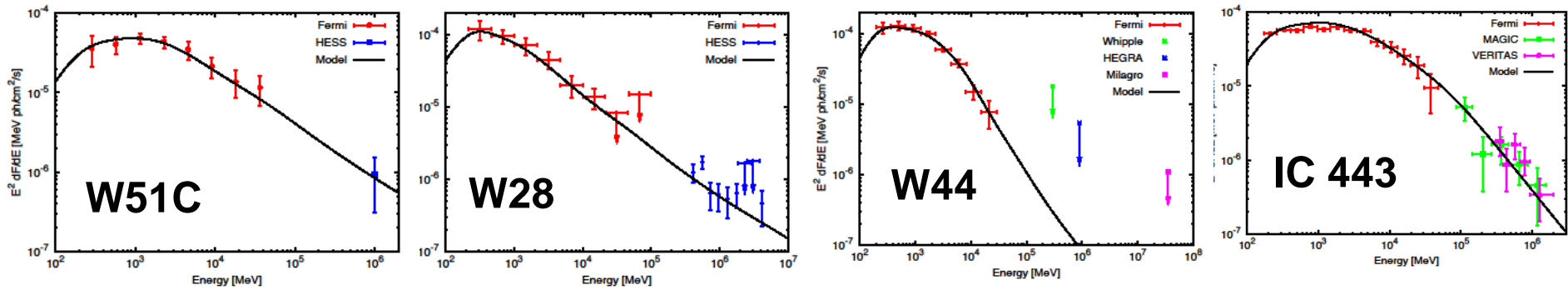
$$D(E) = 10^{28} \times \left[ \frac{E}{10\text{GeV}} \right]^{\delta} \text{cm}^2 \text{s}^{-1}$$

Given  $N_{\text{esc}}(E)$ ,  $t_{\text{esc}}(E)$  and  $R_{\text{esc}}(E)$ , one can obtain  $F(E)$

# Gamma-ray spectra



# Gamma-ray spectra



Our model show that all the runaway CR spectrum of Fermi SNRs are the same spectrum, even though the observed spectra are different.

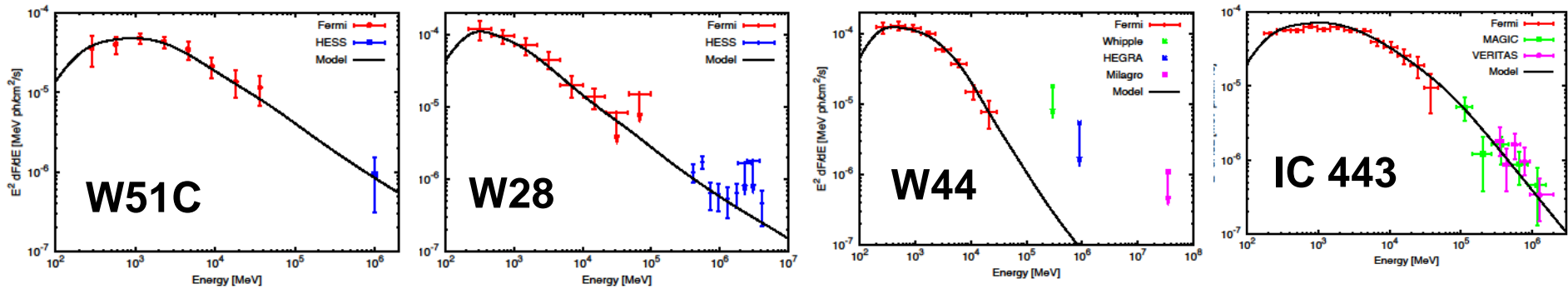
$$s = 2.2 \quad , \quad s_{\text{esc}} = s + \beta / \alpha = 2.38$$

To explain the Galactic CR spectrum,  $Q_{\text{sour}} \propto E^{-2.1-2.4}$

Hence, our results support that

SNRs are indeed the source of Galactic CRs.

# Break at around 10 GeV



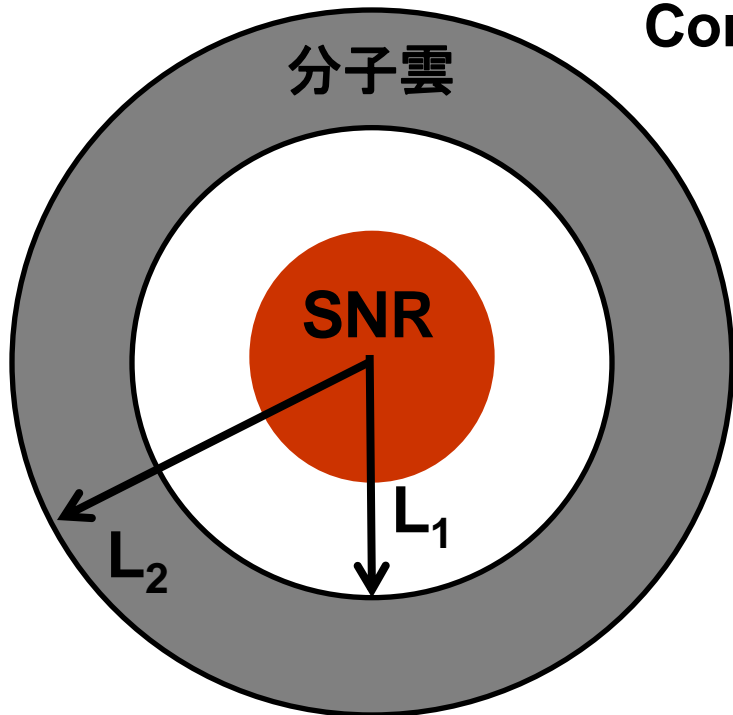
Considering the stellar wind before the SN,

$$L_1 \sim 10 \text{ pc} \quad \text{Weaver et al. (1977)}$$

When  $R_{\text{sh}} = L_1$ , all the CRs escape,

that is,  $E_{\text{max}} = t^{-\alpha}$ ,  $\alpha \rightarrow \infty$

$$s_{\text{esc}} = s + \beta / \alpha \rightarrow s$$





# Summary

Recent observations suggest the magnetic field amplification and that the maximum energy is limited by the escape process.

The runaway CR spectrum is different from that in the accelerator.

$$f_{\text{SNR}} \propto t^{\beta} E^{-s}, E_{\text{max}} \propto t^{-\alpha} \longrightarrow f_{\text{esc}} \propto E^{-s_{\text{esc}}}$$

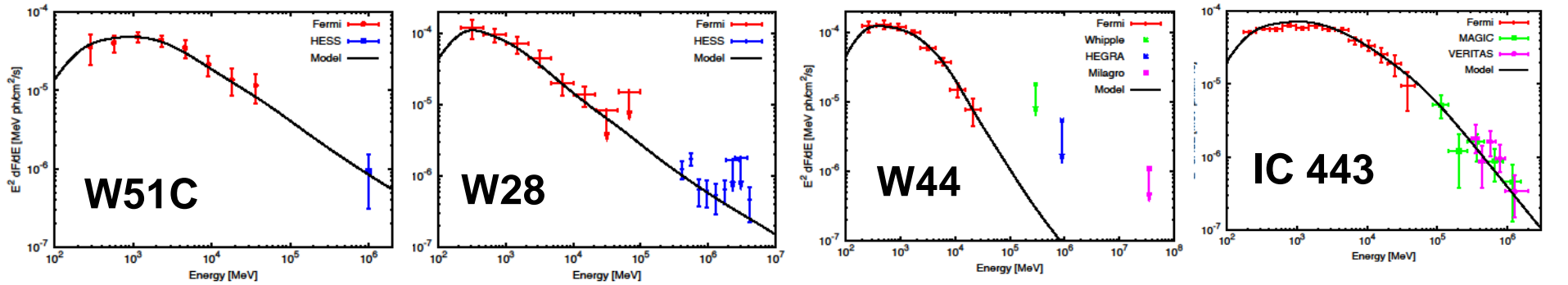
$$s_{\text{esc}} = s + \frac{\beta}{\alpha}$$

Spectra of Fermi SNRs are broken power law and steep.

The spectra can be interpreted as the interaction of runaway CRs with molecular clouds.

All the runaway CR spectra of Fermi SNRs are the same as the spectrum expected from the Galactic CR observation.

# Model Parameters



SNR	$\alpha$	$\beta$	$\kappa$	$s$	$\delta$	$\chi$	$L_1$ (pc)	$L_2$ (pc)	$t_{\text{age}}$ (kyr)	$p_{\text{br},1}$ (GeV/c)	$p_{\text{br},2}$ (GeV/c)	$p_{\text{br},\text{ext}}$ (GeV/c)
W51C	6.5	1.2	0.04	2.2	0.20	0.1	14.7	23.7	31.5	13.1	314	21.9
W28	6.5	1.2	0.04	2.2	0.19	0.9	18.5	26.9	63.0	2.96	< 1.43	< 1.43
W44	6.5	1.2	0.04	2.2	0.40	1.0	12.4	16.2	23.1	39.8	< 1.43	< 1.43
IC 443	6.5	1.2	0.04	2.2	0.62	0.01	11.5	14.7	23.1	62.9	2.93	151